Alternating Currents.

Both in Science and Technology one often encounters voltages and currents varying harmonically in time, that is like a cosine or sine function. Thus consider a voltage given by

$$U(t) = U_0 \cos(\omega t + \phi) \tag{1}$$

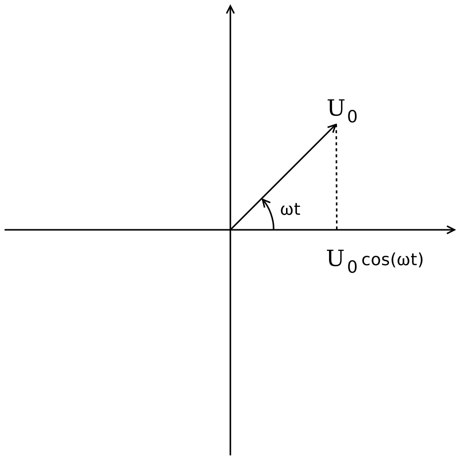
U_0 is the maximum value the voltage can become and is called *the amplitude*. ω is called the *the cyclic frequency*. The harmonic signal is periodic, repeating itself in a time interval T called *the period*. Another concept is *the frequency* ν, the number of periods per time interval. One finds that $\omega = 2\pi\nu = 2\pi/T$. ϕ is called *the phase constant* and is arbitrary in the sense that it will change if the zero point of *the time*, t is changed. The total argument $(\omega t + \phi)$ of the cosine function is occasionally called the *phase*.

It is commonly known, that the electric line we encounter at our homes are alternating currents. It is usually denoted by its voltage, $220V$. This however is not the amplitude but the so-called RMS (root mean square) value, U_{rms}. The RMS-voltage is the magnitude of a corresponding direct current that would deliver the same mean power in an Ohmic resistor.

> Exercise: Show that $U_0 = \sqrt{2}U_{rms}$. Hint: One may use that the mean values of $\cos^2(u)$ and $\sin^2(u)$ both are $1/2$, since $\cos^2(u) + \sin^2(u) = 1$

It is very useful to have a geometric picture of a time-varying harmonic signal. Imagine the signal being the result of a projection on the abscissa of vector in the plane, see figure 1. The vector has the length U_0 and rotates at angular velocity ω being at the angle $\omega t + \phi$ with the abscissa at time t.

> Example: In rural areas where the power supply is still distributed by overhead wires you often see five wires between the masts. One is for street lighting, the next is the neutral conductor (nominally at zero tension) and the remaining three are phase conductors for the consumers along the road. Each represent an individual $220V$ power supply; but

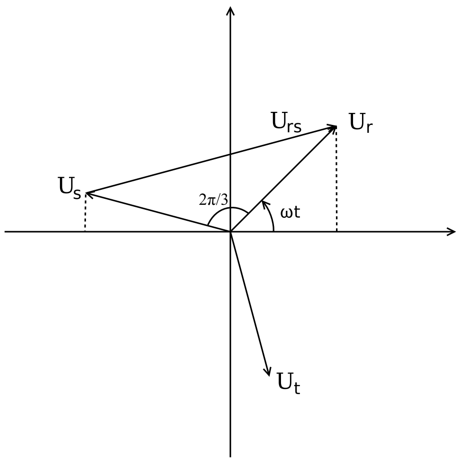


Figur 1 Representation of a harmonic varying voltage by a phasor.

the tensions of the three phase conductors are phase-delayed mutually by one-third of a period. If we name the three wires by r, s and t we have

$$U_r(t) = U_0 \cos(\omega t) \tag{2}$$
$$U_s(t) = U_0 \cos(\omega t + \tfrac{2\pi}{3}) \tag{3}$$
$$U_t(t) = U_0 \cos(\omega t + \tfrac{4\pi}{3}) \tag{4}$$

The three tensions are represented by their phasors in figure 2. One may ask: What is the voltage between two phase conductors? To answer this we have to subtract two harmonic functions with different phases. This can be done geometrically: The difference between the projections of two vectors (phasors) is the same as the projection of the (vectorial) difference between those same vectors. The vector difference is readily seen to have the magnitude $\sqrt{3}\, U_0$ and this is the potential difference between two ohas conductors. The RMS-value scaling in the same way is thus $\sqrt{3}\, 220V$=$380V$.



Figur 2 Phasors of a 3-phase power supply.