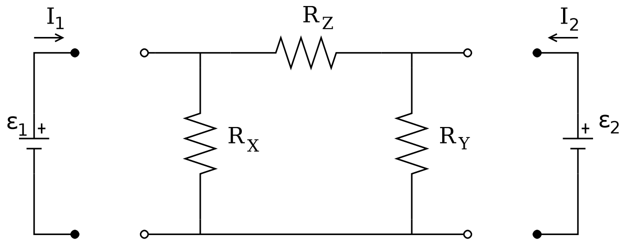
Two-ports

A two-port is a system that can interact with its surroundings via two ports - two energy bonds. For electic networks each bond is realized by two leads. In linear electric network terminology a two-port is thus usually called a four-terminal. As the first example take a look at the so-called Π-network of figure1.



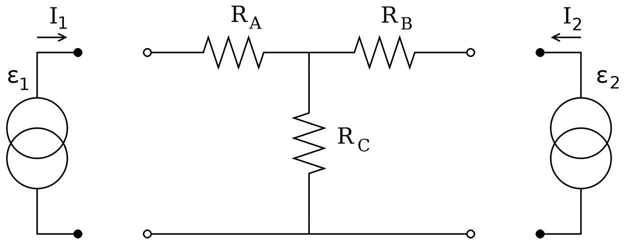
Figur 1 A Π-network driven by voltages.

Imagine two voltage sources ε_1 og ε_2 connected to port 1 og port 2. Since the network is linear we can write up a linear relation between the currents I_1, I_2 and the driving electromotoric forces. The coefficients of this relation we put into a matrix,

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = G_\Pi \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

Exercise 1: Use the laws of Kirchhoff to find the elements of G_Π

As another example we take the so-called T-network of figure 2. In this case we think of the currents I_1 and I_2 being generated from external current sources connected to port 1 and 2. The electromotive forces ϵ_1 and ϵ_2 needed in order



Figur 2 A T-network driven by currents.

to create these currents are again linear in the currents. Thus we write,

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = H_T \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Exercise 2: Calculate the elemnts of H_T by the laws of Kirchhoff.

Exercise 3: A Π-network can be replaced by a T-network. How should one choose R_A, R_B, R_C as a function of R_X, R_Y, R_Z?

Exercise 4: On the other hand, a T-network can be replaced by a Π-network. How should one chose R_X, R_Y, R_Z as a function of R_A, R_B, R_C?

In both of the preceding examples we have set up a relation between a pair of input variables and a pair of output variables. The two input variables consisted of one energy bond variable from each port. The pair of output variables were the corresponding conjugated energy bond variables. Furthermore the orientation of the currents were chosen in a manner that the energy flow is positive towards the system. In such a case the matrix connecting input and ouput is called a response matrix. G og H are both response matrices.

We have the option of conceiving the linear relations in another way. If we consider the two-port as a system transporting energy from port 1 to 2 (and maybe it uses some of the energy itself), then (ε_1, I_1) should be input variables and (ε_2, I_2) be output variables. Furthermore we should switch the orientation of the current of port 2, counting the energy flow positive on leaving port 2 (but still positive entering port 1). The linear relationship is now expressed by the so-called transfermatrix T,

$$\begin{pmatrix} \varepsilon_2 \\ -I_2 \end{pmatrix} = T \begin{pmatrix} \varepsilon_1 \\ I_1 \end{pmatrix}$$

Exercise 5: Calculate the transfer matrices of the Π- and T-network above.

Execise 6: Explain why composition of two-ports correspond to matrixmultiplication of their transfer matrices.

Exercise 7: By Kirchhoff's laws, find the transfer matrices of the two simple two-ports of figure 3.

Exercise 8: Determine T_{Π} and T_T of the Π- and T- network applying the multiplication law and compare with the expressions already found.

Figur 3 Two simple two-ports.