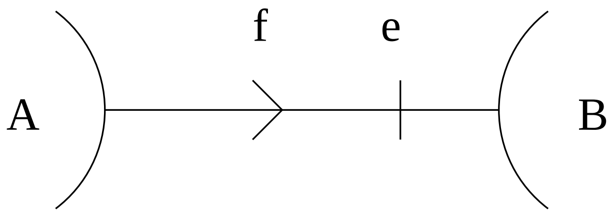
Energy bonds.

Physical phenomenae may often be described in terms of exchange of energy between different systems. We will illustrate the exchange of energy between two systems A and B graphically as in figure 1 with a line connecting the two systems. The line is called an *energy bond*. The two systems interact via the energy bond. The transfered energy pr. time or the power, dE/dt should be given by the product of two variables e og f in order to comply with the energy bond formalism. e is the generalised voltage or force (*effort*) and f is the generalised current or velocity (*flow*). e is said to be conjugated to f and vice versa. In order to tell the current from the voltage one has to

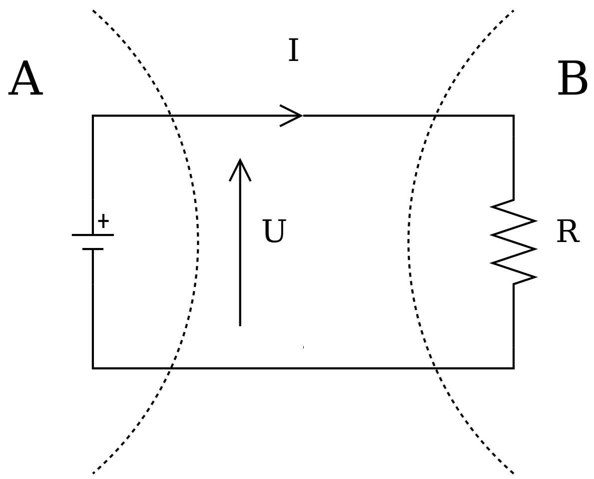


Figur 1 Energetic interaction between two systems.

consider the parity under time reversal. The current changes sign under time reversal while the voltage is unchanged. It is obvious that exactly one of the conjugated variables changes sign under time reversal since their product, the energy current does that. The energy bond may have two spacial directions, from A to B or from B to A. If we reverse the spacial direction of the energy bond but describe the same physical situation, we must concurrently change the sign of the symbolic expression for the energy current. This implies that one of the conjugated variables has to change it sign. It is an axiom - *the flow-orientation rule*- in the energy bond formalism, that it is the current (flow) which changes sign in this case also. We are going to use symbols for effort and flow that reminds us about this: an arrow on the energy bond indicating the flow and a bar indicating the effort. The direction of the flow arrow is the direction we subjectively choose as the positive direction of the energy current. In the following a number of examples of energy bonds are given.

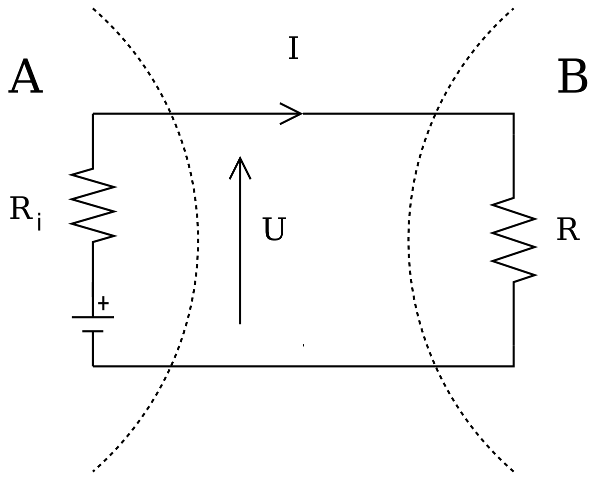
Electric interaction. Consider a battery connected to a resistor as in figure 2. Here the battery is system A and the resistor system B, whereas the two wires connecting the battery and the resistor constitutes the energy bond (there is only one energy bond). e is the voltage U, and f is the current I.

The battery is an *ideal voltage source* in this case delivering a constant voltage regardless of the size of the current that the load demands.



Figur 2 An ideal voltage source connected to a resistor. The two wires constitute the energy bond.

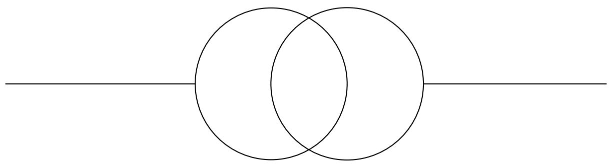
A more realistic situation is shown in figure 3. A real battery is not able to deliver an arbitrary large current and this can be modeled by imagining the battery as builded by an ideal voltage source, ϵ in series with a so-called inner resistance, R_i. The voltage, U measured at the poles of the real battery is now dependent on the current I, flowing since Ohm's law gives $U = \epsilon - R_i I$. The example shows, that voltage and current in an energy bond in general are determined by both systems in union, a fact that is also infered by the word interaction.



Figur 3 An element with an inner resistance connected to an (external) resistance.

Exercise: By different loads of a battery the currents 1A, 5A, 10A og 20A respectively are flowing. The corresponding voltages are measured as 11.8V, 10.7V, 9.5V og 6.8V. Plot the measured voltage as a function of the current. What is the electromotoric force, the short circuit current and the inner resistance?

Corresponding to an ideal voltage source one may also consider an *ideal current source* delivering a constant current unaffected by the voltage nescessary to withstand this current. Such an element has the symbol shown in figure 4.



Figur 4 An ideal current source

Exercise: Explain, how it is possible approximately to construct a current source by a voltage source and a large resistance.

Mechanical interaction.

> Exercise: Explain, that the conjugated variables in mechanical systems are force and velocity.
> Exercise: Explain, that the conjugated variabels in hydrodynamic systems are the pressure and the volume flow.
> Exercise: Explain, that the conjugated variables in rotating mechanical systems are torque and angular velocity.

Reference: Look into Peder Voetmann Christiansen, *Energy Bond Graphs*, IMFUFA-text 419 (2003).