

Modelling Hematopoietic Stem Cells and their Interaction with the Bone Marrow Microenvironment.

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Statistics and Biomathematics Seminar
~~Chalmers University of Technology, Gothenburg, Sweden~~
Online, from my living room
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- ▶ Developing a mathematical model of HSC

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- ▶ Model analysis and results

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- ▶ Background for the work
- ▶ Hematopoietic Stem Cells (HSC)
- ▶ Developing a mathematical model of HSC
- ▶ Model analysis and results
- ▶ A combination of two models and fit to patient data

- ▶ *Cancitis* group at RUC: Mathematical modelling of blood cancers (leukemias), in particular Myeloproliferative Neoplasms (MPNs).



- ▶ *Cancitis* group at RUC: Mathematical modelling of blood cancers (leukemias), in particular Myeloproliferative Neoplasms (MPNs).



- ▶ Part of a larger Danish collaboration. Direct work with clinicians from Zealand University Hospital, Roskilde.



- ▶ *Cancitis* group at RUC: Mathematical modelling of blood cancers (leukemias), in particular Myeloproliferative Neoplasms (MPNs).



- ▶ Part of a larger Danish collaboration. Direct work with clinicians from Zealand University Hospital, Roskilde.



- ▶ Monthly meetings for discussion and sharing of knowledge.

Myeloproliferative Neoplasms and Hematopoietic Stem Cells

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- ▶ **MPNs:** Group of diseases characterized by overproduction of blood cells.

Myeloproliferative Neoplasms and Hematopoietic Stem Cells

- ▶ **MPNs:** Group of diseases characterized by overproduction of blood cells.
- ▶ **Hematopoietic Stem Cells (HSC):** The root of blood production.

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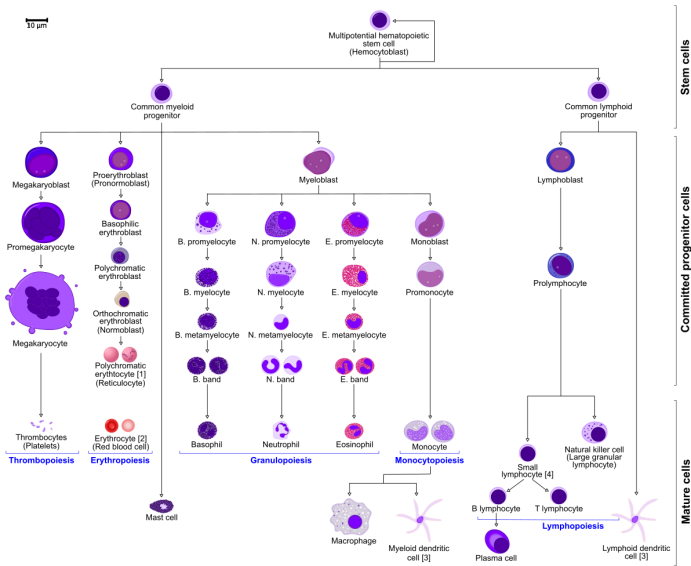
Myeloproliferative Neoplasms and Hematopoietic

Stem cells

Bone marrow

Blood

Tissue



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- ▶ **MPNs:** Group of diseases characterized by overproduction of blood cells.
- ▶ **Hematopoietic Stem Cells (HSC):** The root of blood production.
- ▶ **Leukemic stem cells:** Mutations of HSC can lead to disease.

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- ▶ **Our simplification:** One disease, but different stages.

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 - ▶ Two cell “types”, Healthy and malignant.

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 - ▶ Two cell “types”, Healthy and malignant.
 - ▶ Stem cells and mature blood cells.

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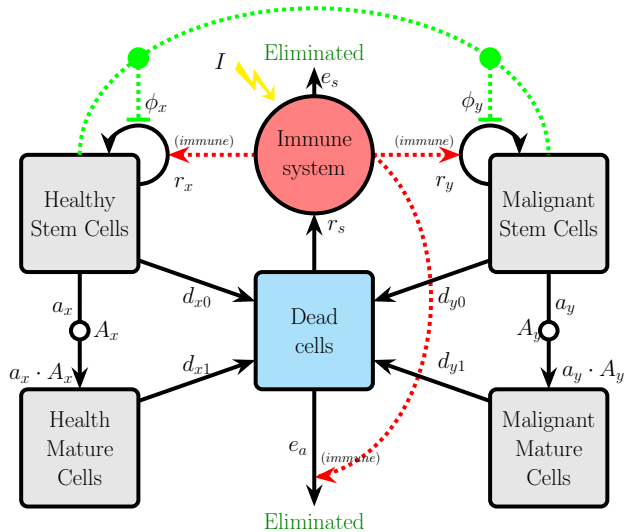
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- ▶ **Our simplification:** One disease, but different stages.
- ▶ **Previous modelling work:**
 - ▶ Two cell “types”, Healthy and malignant.
 - ▶ Stem cells and mature blood cells.
 - ▶ Feedback from the blood through the immune system and inflammation.

The “Cancitis” Model



The "Cancitis" Model

System of 6 ODEs, with 17 parameters.

$$\dot{x}_0 = (r_x \phi_x s - d_{x_0} - a_x) x_0 \quad (1a)$$

$$\dot{x}_1 = a_x A_x x_0 - d_{x_1} x_1 \quad (1b)$$

$$\dot{y}_0 = (r_y \phi_y s - d_{y_0} - a_y) y_0 \quad (1c)$$

$$\dot{y}_1 = a_y A_y y_0 - d_{y_1} y_1 \quad (1d)$$

$$\dot{a} = d_{x_0} x_0 + d_{y_0} y_0 + d_{x_1} x_1 + d_{y_1} y_1 - e_a a s \quad (1e)$$

$$\dot{s} = r_s a - e_s s + I \quad (1f)$$

$$\phi_x = \phi_x(x_0, y_0) = \frac{1}{1 + (c_{xx} x_0 + c_{xy} y_0)} \quad (1g)$$

$$\phi_y = \phi_y(x_0, y_0) = \frac{1}{1 + (c_{yx} x_0 + c_{yy} y_0)} \quad (1h)$$

The “Cancitis” Model

- ▶ A lot of parameters, but most are described in the literature.

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The “Cancitis” Model

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- ▶ Dynamics of model can be related to data from clinics.

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The “Cancitis” Model

- ▶ A lot of parameters, but most are described in the literature.
- ▶ Dynamics of model can be related to data from clinics.
- ▶ Agreement with a large subset of patient-data.



Figure with data excluded from online version

Patient-data from Interferon- α treated patients

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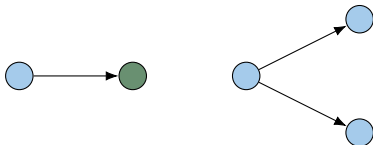
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- ▶ A lot of parameters, but most are described in the literature.
- ▶ Dynamics of model can be related to data from clinics.
- ▶ Agreement with a large subset of patient-data.
- ▶ However, what about the mechanistic interpretation of parameter-changes?

Modelling: Biological assumptions

We wish to include the most important features of HSC.

- ▶ Characterised by *multi-potent differentiation* and *self-renewal*.

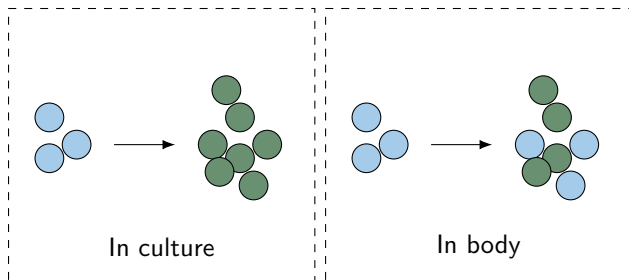


where  are stem cells and  is a differentiated/progenitor cell.

Modelling: Biological assumptions

We wish to include the most important features of HSC.

- ▶ Characterised by *multi-potent differentiation* and *self-renewal*.
- ▶ Lack of self-renewal in culture



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- ▶ Characterised by *multi-potent differentiation* and *self-renewal*.
- ▶ Lack of self-renewal in culture
- ▶ Bone-marrow “niches” interact with HSC in a yet unspecified way.

We wish to include the most important features of HSC.

- ▶ Characterised by *multi-potent differentiation* and *self-renewal*.
- ▶ Lack of self-renewal in culture
- ▶ Bone-marrow “niches” interact with HSC in a yet unspecified way.

Previous mathematical models of HSC behaviour:
(Ashcroft et al., 2017), (Wang, Stiehl et al. 2017),
(Becker et al., 2019), (Wilson and Trumpp, 2006).

Modelling of the HSCs and their niches

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Central hypothesis:

Limited self-renewing division, exhaustion after division.

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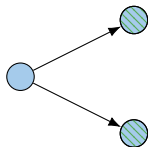
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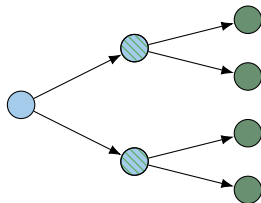
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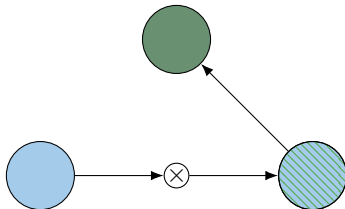
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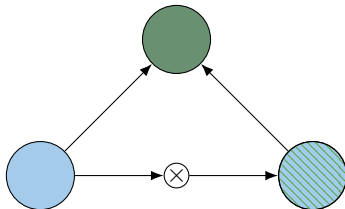
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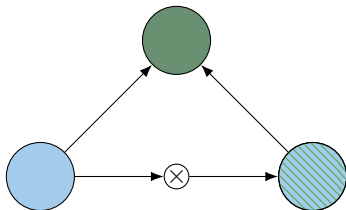
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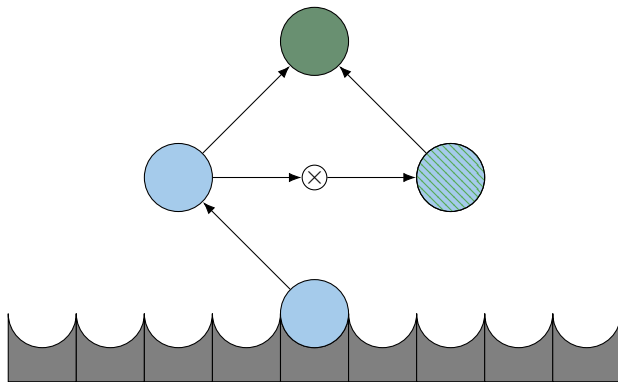
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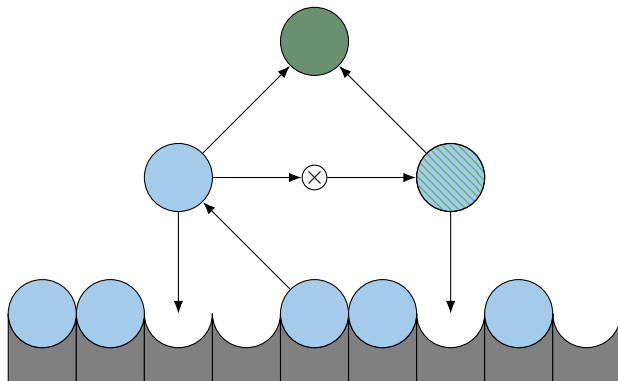
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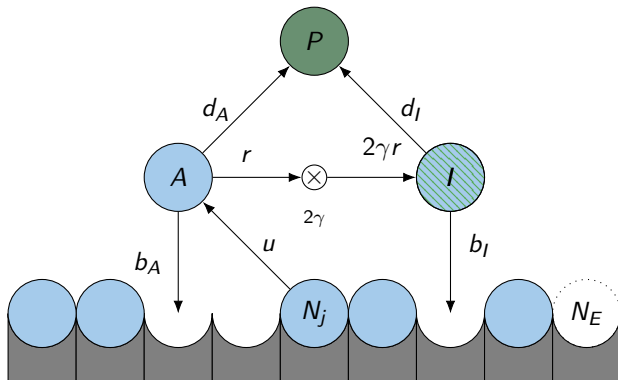
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N_j : Niche-bound, A : Active, I : Inactive, N_E : Empty niches

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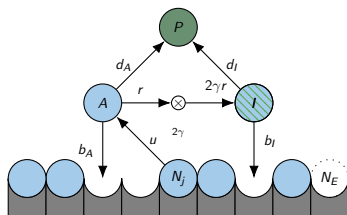
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Modelling of the HSCs and their niches

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$$\frac{dN_E}{dt} = -b_I N_E I - b_A N_E A + u N_j$$

$$\frac{dN_j}{dt} = b_I N_E I + b_A N_E A - u N_j$$

$$\frac{dI}{dt} = 2\gamma r A - b_I N_E I - d_I I$$

$$\frac{dA}{dt} = u N_j - b_A N_E A - r A - d_A A$$

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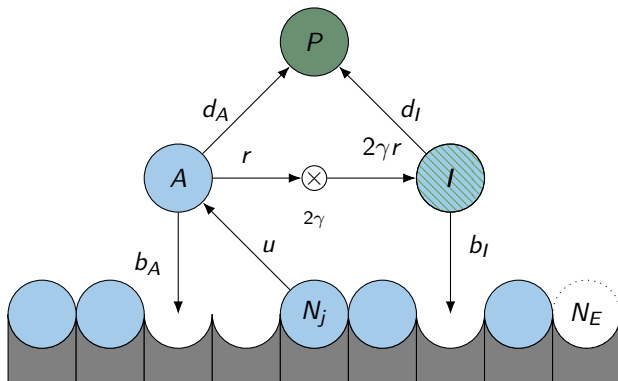
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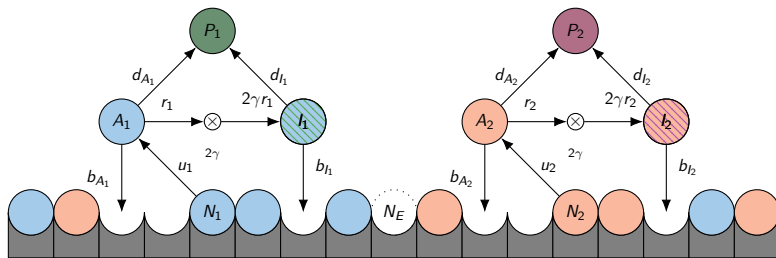
Competition for niche-space



N_j : Niche-bound, A : Active, I : Inactive, N_E : Empty niches

Considering multiple subpopulations of stem cells:
Work of Thomas Stiehl \Rightarrow Healthy and malignant cells
compete for a shared niche.

Competition for niche-space



$$\frac{dN_E}{dt} = u_1 N_1 + u_2 N_2 - N_E (b_{I_1} I_1 + b_{A_1} A_1 + b_{I_2} I_2 + b_{A_2} A_2)$$

$$\frac{dN_1}{dt} = b_{I_1} N_E I_1 + b_{A_1} N_E A_1 - u_1 N_1$$

$$\frac{dN_2}{dt} = b_{I_2} N_E I_2 + b_{A_2} N_E A_2 - u_2 N_2$$

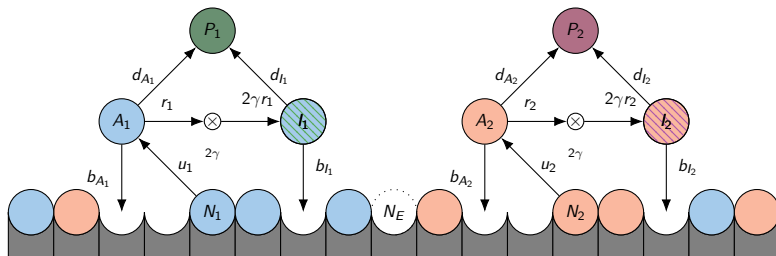
$$\frac{dI_1}{dt} = 2\gamma r_1 A_1 - b_{I_1} N_E I_1 - d_{I_1} I_1$$

$$\frac{dI_2}{dt} = 2\gamma r_2 A_2 - b_{I_2} N_E I_2 - d_{I_2} I_2$$

$$\frac{dA_1}{dt} = u_1 N_1 - (b_{A_1} N_E + r_1 + d_{A_1}) A_1$$

$$\frac{dA_2}{dt} = u_2 N_2 - (b_{A_2} N_E + r_2 + d_{A_2}) A_2$$

Competition for niche-space



$$\frac{dN_E}{dt} = \sum_{i=1}^n u_i N_i - N_E \sum_{i=1}^n (b_{I_i} l_i + b_{A_i} A_i)$$

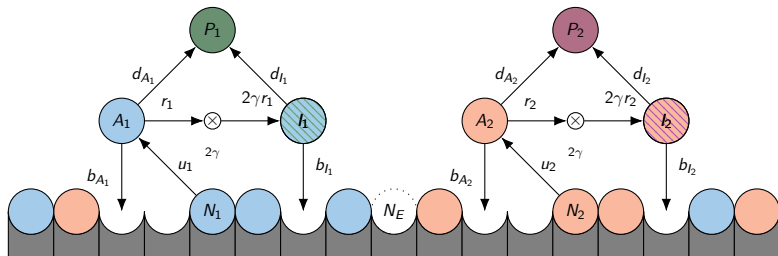
$$\frac{dN_j}{dt} = b_{I_j} N_E l_j + b_{A_j} N_E A_j - u_j N_j$$

$$\frac{dl_j}{dt} = 2\gamma r_j A_j - b_{I_j} N_E l_j - d_{I_j} l_j$$

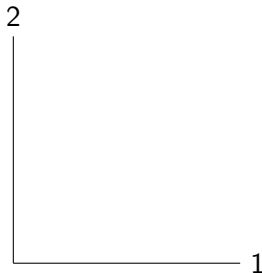
$$\frac{dA_j}{dt} = u_j N_j - b_{A_j} N_E A_j - r_j A_j - d_{A_j} A_j$$

for j from 1 to n , where n is the number of distinct clones.

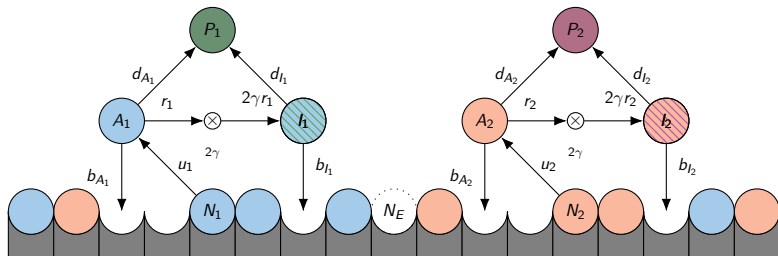
Competition for niche-space



Steady states:

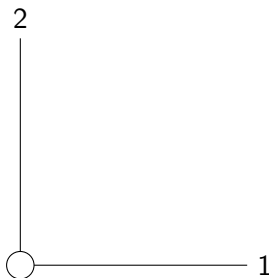


Competition for niche-space



Steady states:

- No cells (Exhaustion)



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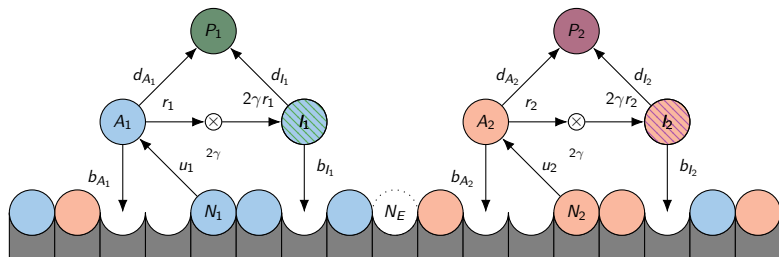
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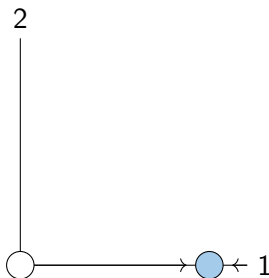
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Competition for niche-space



Steady states:

- ▶ No cells (Exhaustion)
- ▶ Only clone 1 (Health)



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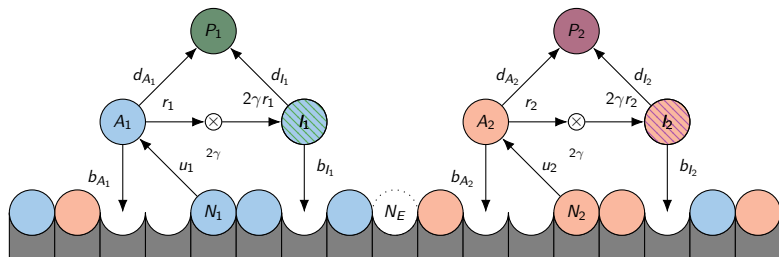
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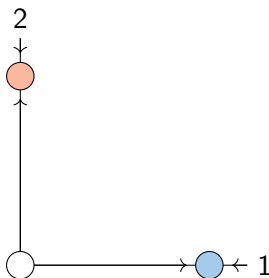
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Competition for niche-space



Steady states:

- ▶ No cells (Exhaustion)
- ▶ Only clone 1 (Health)
- ▶ Only clone 2 (Disease)



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Rasmus Kristoffer
Pedersen

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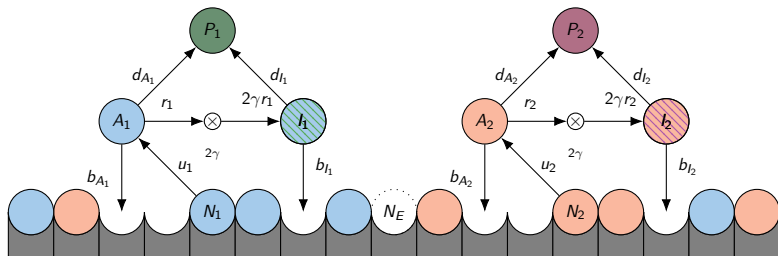
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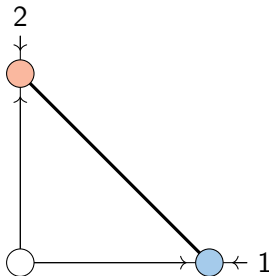
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Competition for niche-space



Steady states:

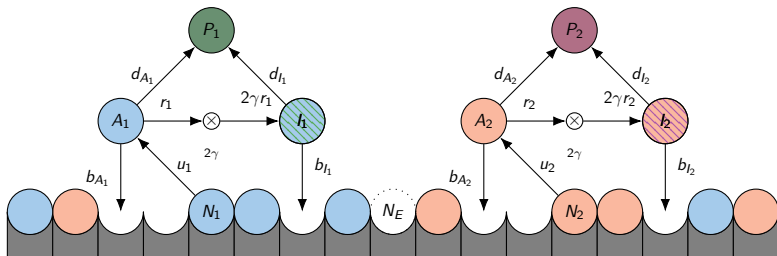
- ▶ No cells (Exhaustion)
- ▶ Only clone 1 (Health)
- ▶ Only clone 2 (Disease)
- ▶ Co-existence (Suppressed disease?)



Competition for niche-space - Fitness

Stability of steady states depends on HSC fitness:

$$F_1 = \frac{b_1 (r_1 - d_{A_1})}{d_{I_1} (r_1 + d_{A_1})} \quad \text{and} \quad F_2 = \frac{b_2 (r_2 - d_{A_2})}{d_{I_2} (r_2 + d_{A_2})}$$

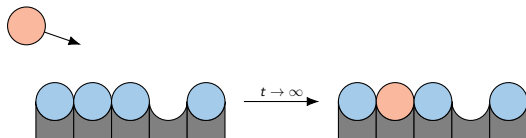


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- If $F_1 = F_2$ then coexistence is possible.



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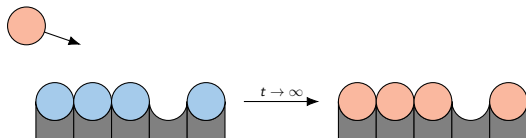
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- ▶ If $F_1 < F_2$ then clone 2 outcompetes clone 1

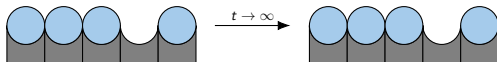


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- ▶ If $F_1 < F_2$ then clone 2 outcompetes clone 1
- ▶ If $F_1 > F_2$ then clone 1 outcompetes clone 2



What do we have so far?

- ▶ Mathematical model describing central mechanisms of HSCs.

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- ▶ A notion of stem cell fitness.

Which is surprisingly similar to fitness as found in ecological systems!

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Article draft almost ready for submission!

Under certain (biological) assumptions, we can reduce the model.

$$\dot{N}_E = \sum_{i=1}^n u_i N_i - N_E \sum_{i=1}^n (b_{l_i} l_i + b_{A_i} A_i)$$

$$\dot{N}_j = b_{l_j} N_E l_j + b_{A_j} N_E A_j - u_j N_j$$

$$\dot{l}_j = 2\gamma r_j A_j - b_{l_j} N_E l_j - d_{l_j} l_j$$

$$\dot{A}_j = u_j N_j - b_{A_j} N_E A_j - r_j A_j - d_{A_j} A_j$$

where $\dot{} = \frac{d}{dt}$ and j from 1 to n .

Since $\dot{N}_E + \sum_{i=1}^n \dot{N}_i = 0$, we set $N_E = K - \sum_{i=1}^n N_i$:

$$\dot{N}_j = (b_{l_j} l_j + b_{A_j} A_j) \left(K - \sum_{i=1}^n N_i \right) - u_j N_j$$

$$\dot{l}_j = 2\gamma r_j A_j - \left(b_{l_j} \left(K - \sum_{i=1}^n N_i \right) + d_{l_j} \right) l_j$$

$$\dot{A}_j = u_j N_j - \left(b_{A_j} \left(K - \sum_{i=1}^n N_i \right) + r_j + d_{A_j} \right) A_j$$

Assuming $b_{A_j} = 0$:

$$\dot{N}_j = b_{I_j} I_j \left(K - \sum_{i=1}^n N_i \right) - u_j N_j$$

$$\dot{I}_j = 2\gamma r_j A_j - \left(b_{I_j} \left(K - \sum_{i=1}^n N_i \right) + d_{I_j} \right) I_j$$

$$\dot{A}_j = u_j N_j - (r_j + d_{A_j}) A_j$$

Scaling variables:

$$\dot{N}_j = u_j \left(1 - \sum_{i=1}^n N_i \right) I_j - u_j N_j$$

$$\dot{I}_j = 2\gamma b_{I_j} K A_j - \left(b_{I_j} K \left(1 - \sum_{i=1}^n N_i \right) + d_{I_j} \right) I_j$$

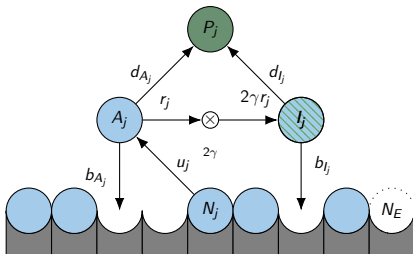
$$\dot{A}_j = r_j N_j - (r_j + d_{A_j}) A_j$$

Reducing the model

Consequences of biological assumptions:

If $N_j \gg N_E$, then $\dot{I}_j \approx 0$

If $N_j \gg (I_j + A_j)$, then $\dot{A}_j \approx 0$



Quasi-steady-state approximation, $\dot{I}_j = 0$ yields:

$$\dot{N}_j = \frac{2\gamma u_j (1 - \sum_{i=1}^n N_i)}{\alpha_j + 1 - \sum_{i=1}^n N_i} \left(1 - \sum_{i=1}^n N_i \right) A_j - u_j N_j$$

$$\dot{A}_j = r_j N_j - (r_j + d_{A_j}) A_j$$

where $\alpha_j = \frac{d_j}{b_j K}$.

Quasi-steady-state approximation, $\dot{A}_j = 0$ yields:

$$\dot{N}_j = u_j \left(\frac{2\gamma\rho_j (1 - \sum_{i=1}^n N_i)}{\alpha_j + 1 - \sum_{i=1}^n N_i} - 1 \right) N_j$$

where $\alpha_j = \frac{d_{I_j}}{b_{I_j}K}$ and $\rho_j = \frac{r_j}{r_j + d_{A_j}}$.

The reduced HSC-niche model

- ▶ Same asymptotic behaviour as full model.

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The reduced HSC-niche model

- ▶ Same asymptotic behaviour as full model.
- ▶ Captures the slow dynamics of the full model.

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The reduced HSC-niche model

- ▶ Same asymptotic behaviour as full model.
- ▶ Captures the slow dynamics of the full model.
- ▶ Can be written in term of the fitness:

$$\dot{N}_j = \frac{u_j \alpha_j F_j N_j}{\alpha_j + 1 - \sum_{i=1}^n N_i} \left(1 - \sum_{i=1}^n N_i - F_j^{-1} \right)$$

- ▶ Same asymptotic behaviour as full model.
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- ▶ Possibility for further simplification and simple analysis when $n = 2$

Putting it all together

Cancitis model

$$\dot{x}_0 = (r_x \phi_x s - d_{x0} - a_x) x_0$$

$$\dot{x}_1 = a_x A_x x_0 - d_{x1} x_1$$

$$\dot{y}_0 = (r_y \phi_y s - d_{y0} - a_y) y_0$$

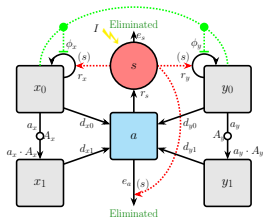
$$\dot{y}_1 = a_y A_y y_0 - d_{y1} y_1$$

$$\dot{a} = d_{x0} x_0 + d_{y0} y_0 + d_{x1} x_1 + d_{y1} y_1 - e_a a s$$

$$\dot{s} = r_s a - e_s s + I$$

$$\phi_x = \phi_x(x_0, y_0) = \frac{1}{1 + (c_{xx} x_0 + c_{xy} y_0)}$$

$$\phi_y = \phi_y(x_0, y_0) = \frac{1}{1 + (c_{yx} x_0 + c_{yy} y_0)}$$



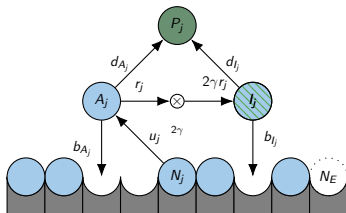
HSC niche model

$$\dot{N}_j = u_j \left(\frac{2\gamma \rho_j (1 - \sum_{i=1}^n N_i)}{\alpha_j + 1 - \sum_{i=1}^n N_i} - 1 \right) N_j$$

where $\alpha_j = \frac{d_{lj}}{b_{lj} K}$ and $\rho_j = \frac{r_j}{r_j + d_{A_j}}$.

and production of progenitors:

$$P_j = d_{A_j} A_j + d_{l_j} I_j$$



Putting it all together

$$\dot{N}_x = u_x s \left(\frac{2\gamma\rho_x(1 - N_x - N_y)}{\alpha_x + 1 - N_x - N_y} - 1 \right) N_x$$

$$\dot{M}_x = A_x P_x - d_{x1} M_x$$

$$\dot{N}_y = u_y s \left(\frac{2\gamma\rho_y(1 - N_x - N_y)}{\alpha_y + 1 - N_x - N_y} - 1 \right) N_y$$

$$\dot{M}_y = A_y P_y - d_{y1} M_y$$

$$\dot{a} = d_{x1} M_x + d_{y1} M_y - e_a a s$$

$$\dot{s} = r_s a - e_s s + I$$

where $P_j = \left(d_{Aj} + \frac{2\gamma r_j}{1 + d_j^{-1} b_j (K - N_x - N_y)} \right) \frac{u_j}{r_j + d_{Aj}} N_j$

- ▶ Perturbation of parameters during treatment

$$\theta(t) = (1 + \delta_{\theta}D(t))\theta(0) \quad (1)$$

where $D(t)$ is blood-level of drug, and δ_{θ} is patient-specific.

- ▶ Perturbation of parameters during treatment

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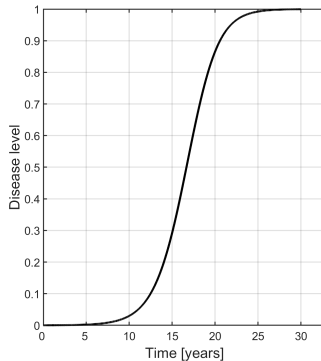
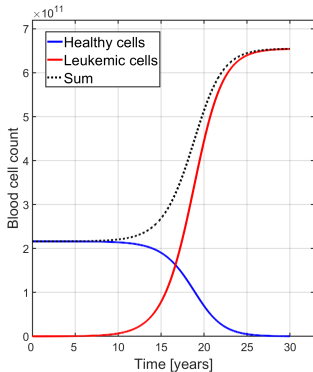
- ▶ Relating measurement from blood samples to the model
 - ▶ Blood-cell count: $M_x + M_y$
(Thrombocytes or leukocytes)
 - ▶ Disease burden in blood: $\frac{M_y}{M_x + M_y}$

Examples of patient-specific fits

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No treatment



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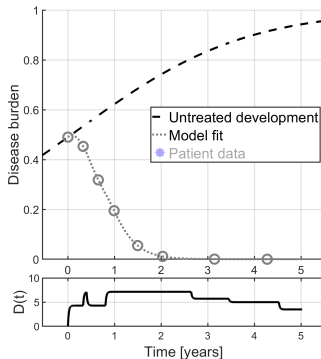
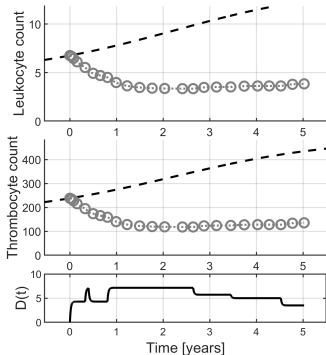
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Examples of patient-specific fits

Patient 1



$\theta(t) = (1 + \delta_\theta D(t))\theta(0)$, fitting only the clearing-rates of mature cells ($\delta_{d_{x_1}} = \delta_{d_{y_1}}$) and the differentiation of active HSC ($\delta_{d_{A_y}}$). Data removed in online version.

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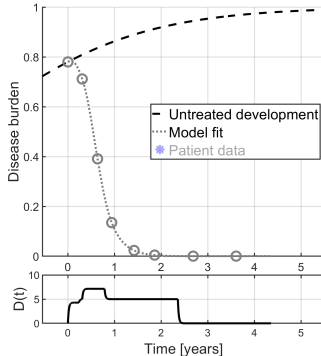
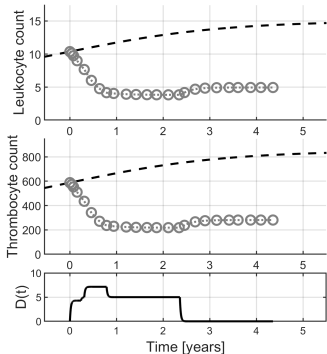
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Examples of patient-specific fits

Patient 2



$\theta(t) = (1 + \delta_\theta D(t))\theta(0)$, fitting only the clearing-rates of mature cells ($\delta_{d_{x_1}} = \delta_{d_{y_1}}$) and the differentiation of active HSC ($\delta_{d_{A_y}}$). Data removed in online version.

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- ▶ Mathematical modelling can help us understand the dynamics of HSC.

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- ▶ Mathematical modelling can help us understand the dynamics of HSC.
- ▶ Limited self-renewal refreshed through niche-interaction leads to a notion of HSC fitness.

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- ▶ Mathematical modelling can help us understand the dynamics of HSC.
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- ▶ Model reduction results in a simpler ODE for HSC, and identifies parameters that are hard to observe.

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- ▶ Combining the simplified HSC niche-model with the Cancitis model yields a model which allows for improved biological interpretation.

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- ▶ Model reduction results in a simpler ODE for HSC, and identifies parameters that are hard to observe.
- ▶ Combining the simplified HSC niche-model with the Cancitis model yields a model which allows for improved biological interpretation.
- ▶ The combined model shows great promise for patient-specific predictions based on treatment dosage.

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Thank you for your attention.

Any questions?



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