Exercises for Lecture 25 (Quantum Computing II), Tuesday, May 16, 2006.

1. Griffiths problem 12.1[New] (Essentially, show that $\alpha| \uparrow\downarrow\rangle + \beta| \downarrow\uparrow\rangle$ cannot be written as a product of two one-spin states).

2. Since the identity matrix and the three Pauli spin matrices are Hermitian, so is any real linear combination of them ($\alpha I + \beta X + \gamma Y + \delta Z$, with $\alpha, \beta, \gamma, \delta$ real). For an arbitrary Hermitian matrix with elements $Q_{11}, Q_{12}, Q_{21}, Q_{22}$, write down the corresponding coefficients $\alpha, \beta, \gamma, \delta$ (don't forget what conditions the Qs need to satisfy for a Hermitian matrix). The Pauli matrices are given in section 4.4.1 (Spin-1/2) of the book (where they are called σ_x etc)

3. The Bell basis states for a two-qubit system are defined as $|b_1\rangle = \frac{1}{\sqrt{2}}(| \uparrow\uparrow\rangle + | \downarrow\downarrow\rangle)$, $|b_2\rangle = \frac{1}{\sqrt{2}}(| \uparrow\uparrow\rangle - | \downarrow\downarrow\rangle)$, $|b_3\rangle = \frac{1}{\sqrt{2}}(| \uparrow\downarrow\rangle + | \downarrow\uparrow\rangle)$, $|b_4\rangle = \frac{1}{\sqrt{2}}(| \uparrow\downarrow\rangle - | \downarrow\uparrow\rangle)$.

   Write the matrix that expresses these states in terms of the "computational basis" states (the product states $| \uparrow\uparrow\rangle$, $| \uparrow\downarrow\rangle$, etc. Be sure to specify in what order you label the computational basis states)

4. Prove the identities $HXH = Z$ and $HZH = X$, where H is the Hadamard gate (unitary operator/matrix) $\frac{1}{\sqrt{2}}(X + Z)$

5. In the reading for lecture 25, from Chapter 8 of Stolze and Suter, page 108, the matrix implementations of the four possible 1-bit to 1-bit functions are given. The first one is dervied. Prove the last three in a similar manner.