Roskilde Universitet

Dybedemoduleksamen i elektrodynamik

Friday 30. January, 2009. 10.00-14.00

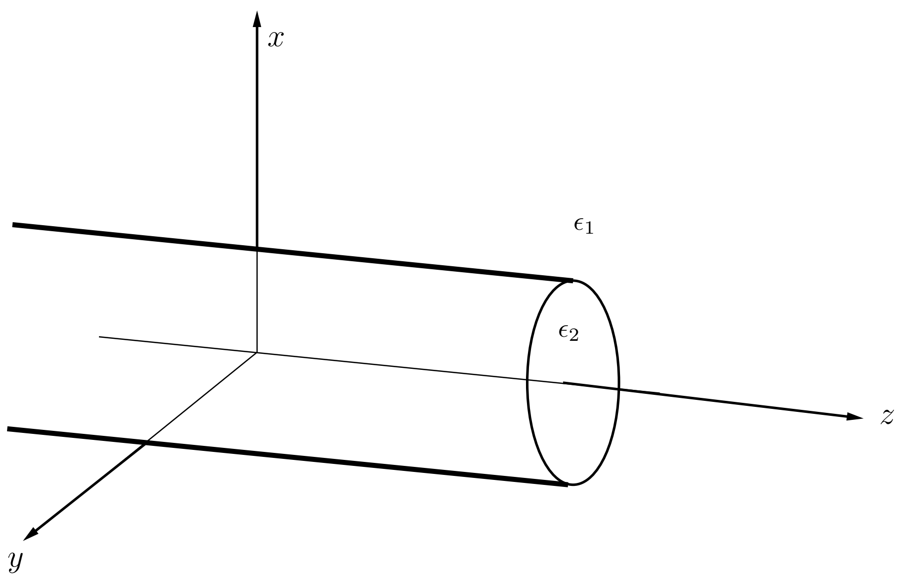
ONLY PERSONAL HELP-MATERIALS ALLOWED: NO COMMUNICATION WITHIN OR OUT FROM THE EXAM ROOM

Question 1 and Question 2 are independent. Each subquestion is weighted equally.

This exam has three pages.

Question 1

A homogeneous displacement field, $\mathbf{D_0} = \mathbf{D_0}\hat{\mathbf{x}}$, is applied along the x-axis in a linear dielectric material with dielectric constant ϵ_1. (Imagine that the dielectric material extends to infinity in all directions). An infinitely long dielectric cylindrical rod of radius R and the dielectric constant ϵ_2 is then placed with its axis along the z-axis perpendicular to the D-field.



Use cylindrical coordinates (s, ϕ, z) in the following.

1 The electrostatic potential, V, is given by Laplace's equation both outside and inside the cylindrical rod. Explain why.

2 Determine the boundary conditions for the electrostatic potential.

3 Show that the electrostatic potential is given by:

$$V(s,\phi) = V_1(s,\phi) = -E_0 s\cos(\phi) + \frac{R^2 E_0}{s}\left(\frac{-2}{\frac{\epsilon_2}{\epsilon_1}+1}+1\right)\cos\phi \quad \text{for} \quad s > R$$

$$V(s,\phi) = V_2(s,\phi) = \frac{-2E_0}{\frac{\epsilon_2}{\epsilon_1}+1} s\cos(\phi) \quad \text{for} \quad s < R$$

where $E_0 = \frac{D_0}{\epsilon_1}$

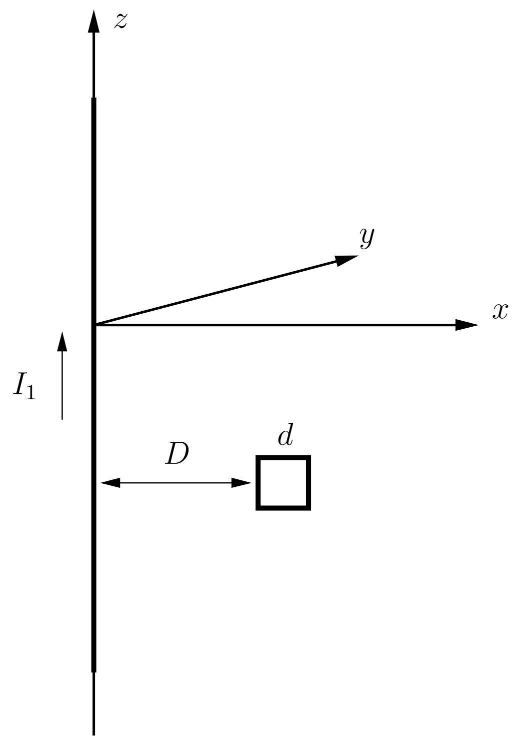
4 Find the bound surface charge (polarization charge) density at the surface of the cylinder.

5 Find the electric field inside the cylinder.

6 Answer questions 4 and 5 in the special case where $\epsilon_1 = \epsilon_2$. Is the result you have obtained physically consistent?

7 Which physical system corresponds electrtostatically to the limit $\epsilon_2 \to \infty$? Discuss it briefly.

Question 2



A long straight wire runs along the z-axis. A small square loop of wire is place in the z-x-plane a distance D away from the wire as shown in the figure above. The sides of the loop have length d and $d \ll D$.

For time $t < 0$ the electrical current in the straight wire is zero. At time $t = 0$ a current $I_1(t)$ with a constant growth rate k starts flowing in the wire, such that for $t > 0$ we have $I_1(t) = kt$.

1 Determine the magnetic field, $\mathbf{B}_1(x, y, z, t)$, generated by the current in the straight wire.

2 Determine the electromotive force generated in the small loop by the magnetic field from question 1.

3 The small loop has an ohmic resistance R and a self inductance L. Find the current $I_2(t)$ generated in the small loop, and show that it grows towards a limiting value. Make a sketch to show the direction of this current.

In the following three sub-questions we study the system at infinite time, $t \to \infty$, where the current $I_2(t)$ has reached its limiting value.

4 Determine the magnetic dipole moment of the loop.

5 Determine the torque on the loop and comment on the stability of the orientation.

6 Determine the force on the loop.