Roskilde Universitet

Dybedemoduleksamen i elektrodynamik

Tuesday 24. February, 2009. 10.00-14.00

ONLY PERSONAL HELP-MATERIALS ALLOWED: NO COMMUNICATION WITHIN OR OUT FROM THE EXAM ROOM

Question 1 and Question 2 are independent. Each subquestion is weighted equally.

This exam has 2 pages.

Question 1

An electromagnetic wave is described by (in spherical coordinates).

$$\begin{aligned}
\mathbf{E}(r,\theta,\phi,t) &= E_0\frac{\sin\theta}{r}\left[\cos(kr-\omega t)-\frac{1}{kr}\sin(kr-\omega t)\right]\hat{\boldsymbol{\phi}} \\
\mathbf{B}(r,\theta,\phi,t) &= \frac{2E_0\cos\theta}{\omega r^2}\left[\sin(kr-\omega t)+\frac{1}{kr}\cos(kr-\omega t)\right]\hat{\mathbf{r}} \\
&\quad +\frac{E_0\sin\theta}{\omega r}\left[\left(\frac{1}{kr^2}-k\right)\cos(kr-\omega t)+\frac{1}{r}\sin(kr-\omega t)\right]\hat{\boldsymbol{\theta}}
\end{aligned}$$

1 Draw a sketch that illustrates the propagation of the wavefronts. Indicate the direction of the electrical field on the sketch.

For notational convenience, let $(kr-\omega t)\equiv u$ and note that the chain rule gives $\frac{\partial}{\partial r}\cos u=-k\sin u$, $\frac{\partial}{\partial r}\sin u=k\cos u$, $\frac{\partial}{\partial t}\cos u=\omega\sin u$, and $\frac{\partial}{\partial t}\sin u=-\omega\cos u$.

2 Show that Gauss's law is obeyed.

3 Show that Faraday's law is obeyed.

4 Which other Maxwell equations must be obeyed?

5 Find the Poynting vector $\mathbf{S}$.

6 By averaging, $\mathbf{S}$, over a full cycle one can find the intensity which is time independent and is given by : $I=\langle\mathbf{S}\rangle=\frac{E_0^2k\sin^2\theta}{2\mu_0\omega r^2}\hat{\mathbf{r}}$. Comment on the r-dependence and the direction seen in this expression.

Question 2

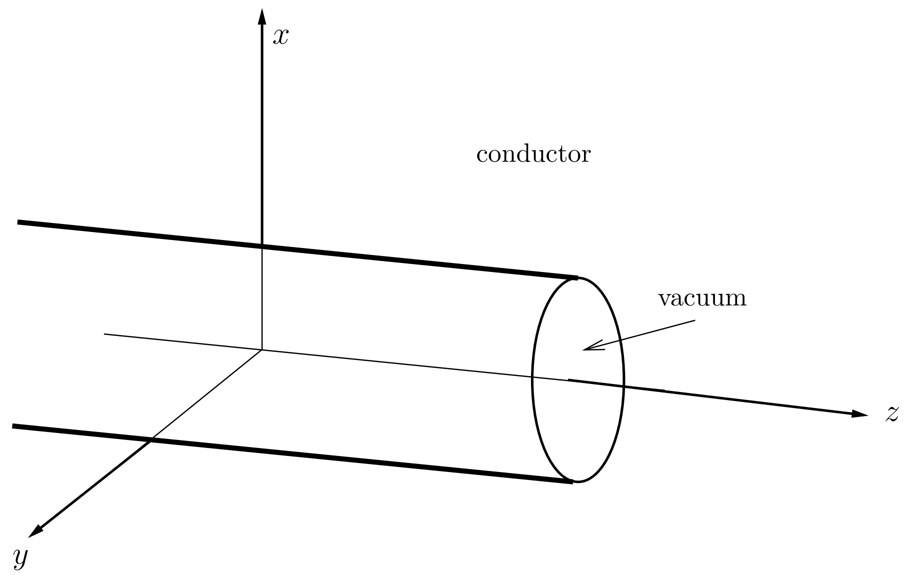
There is an infinitely long cylindrical hole of radius R in the conductor with its axis along the z-axis perpendicular to the E-field. (Imagine that the conductor extends to infinity in all directions).

There is an electric field, which is given by $\mathbf{E}_0 = E_0\hat{\mathbf{x}}$ far away from the cylinder.

A stationary current is flowing in the conductor. Because the conductor is ohmic there is proportionality between the electric field and the current density: $\mathbf{j} = g\mathbf{E}$.

The permittivity and permeability both inside the conductor and in the cylindrical hole have the vacuum values, ϵ_0 and μ_0.

It is in most cases easiest to use cylindrical coordinates (s, ϕ, z) in the following.



1 Show that $\nabla \cdot \mathbf{j} = 0$ in the conductor.

2 The electrostatic potential, V, is given by Laplace's equation both outside and inside the cylindrical hole. Explain why.

3 Determine the boundary conditions for the electrostatic potential.

4 Show that the electrostatic potential is given by:

$$\begin{aligned} V(s,\phi) &= V_1(s,\phi) = -E_0 s\cos(\phi) - E_0\frac{R^2}{s}\cos(\phi) \quad s > R \\ V(s,\phi) &= V_2(s,\phi) = -2E_0 s\cos(\phi) \quad \text{for} \quad s < R \end{aligned}$$

5 Find the electric field inside the cylinder and in the conductor.

6 Find the surface charge density at the surface of the cylinder.

7 Make a sketch of the charge and the electrical field in the x-y plane.