Golden sheet E2009, ver. 1.0

1 Termodynamik

Termodynamikkens 1. lov:	$dU = q + w = q - pdV$
Termodynamikkens 2. lov (isoleret system):	$\Delta S \geq 0\,, \qquad S = k \ln \Omega$
Termodynamikkens 3. lov:	$T \to 0 \Rightarrow S \to 0, C_x \to 0$
Entropitilvækst ved tilført varme:	$dS \geq \frac{q}{T}$
	$dS = \frac{q}{T}$ (Quasistatisk, Reversibelt)
Ækvipartitionsprincippet:	$U = \frac{f}{2} N k_B T$, f = # frihedsgrader.

Termodynamiske potentialer:

Potentiale	Differentialform	Maxwell relation
U	$dU = +TdS - PdV + \mu dN$	$\left(\frac{\partial T}{\partial V}\right)_{S,N} = -\left(\frac{\partial P}{\partial S}\right)_{V,N}$
$H = U + PV$	$dH = +TdS + VdP + \mu dN$	$\left(\frac{\partial T}{\partial P}\right)_{S,N} = +\left(\frac{\partial V}{\partial S}\right)_{P,N}$
$F = U - TS$	$dF = -SdT - PdV + \mu dN$	$\left(\frac{\partial S}{\partial V}\right)_{T,N} = +\left(\frac{\partial P}{\partial T}\right)_{V,N}$
$G = U + PV - TS = \mu N$	$dG = -SdT + VdP + \mu dN$	$\left(\frac{\partial S}{\partial P}\right)_{T,N} = -\left(\frac{\partial V}{\partial T}\right)_{P,N}$

Linære responsefunktioner (dN=0):

Varmekapacitet	$C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$	$C_P = \left(\frac{\partial H}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P$
Udvidelseskoefficient	$\alpha_P = +\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$	$\alpha_S = -\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_S$
Kompressibilitet	$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$	$\kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$
Bulkmodul	$K_T = -V\left(\frac{\partial P}{\partial V}\right)_T = \frac{1}{\kappa_T}$	$K_S = -V\left(\frac{\partial P}{\partial V}\right)_S = \frac{1}{\kappa_S}$
Trykkoefficient	$\beta_V = \left(\frac{\partial P}{\partial T}\right)_V$	$\beta_S = \left(\frac{\partial P}{\partial T}\right)_S$

$dV = 0$: Isochor; $dP = 0$: Isobar; $dT = 0$: Isoterm; $dS = 0$: Adiabat

2 Statistisk mekanik

I termisk ligevægt med omgivelserne: $p(s) = \frac{e^{-\beta E(s)}}{Z}, \quad \beta \equiv \frac{1}{k_B T}$
Tilstandssummen: $Z = \sum_s e^{-\beta E(s)} \quad (\Rightarrow \sum_s p(s) = 1)$
Middelværdi af variablen A: $\langle A \rangle = \sum_s A(s) p(s) = \sum_s A(s) \frac{e^{-\beta E(s)}}{Z}$
Middelenergi: $U = \langle E \rangle = -\frac{\partial \ln(Z)}{\partial \beta} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$
Helmholtz fri energi: $F = -k_B T \ln(Z)$
Skelnelige, uafhængige “partikler”: $Z = Z_1 \cdot Z_2 \cdot Z_3 \cdot Z_4 \cdot ...$ $\quad (E = E_1 + E_2 + E_3 + E_4 + ...)$
Uskelnelige, uafhængige “partikler”: $Z = Z_1^N / N!$

3 Ideal-gas

Tilstandsligning: $pV = Nk_B T = nRT$
Adiabat: $pV^\gamma = const$, $\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$
Isoterm kompression: $W = NkT \ln \frac{V_f}{V_i}$

4 Matematik

Binomial-koefficienten: $\binom{N}{n} = \frac{N!}{n!(N-n)!}$
Stirling: $\ln(N!) \approx N\ln(N) - N \quad (N >> 1)$
Taylor udvikling: $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + ...$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$
$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ...$
$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ...$
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + ...$
$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n$, hvor $\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)...(\alpha-n+1)}{n!}$

$\int_0^\infty x^n \exp(-ax^2) = I(n,a), \quad \frac{\partial I(n,a)}{\partial a} = -I(n+2,a)$		
$I(0,a) = \frac{1}{2}\sqrt{\pi}a^{-1/2}$	$I(2,a) = \frac{1}{4}\sqrt{\pi}a^{-3/2}$	$I(4,a) = \frac{3}{8}\sqrt{\pi}a^{-5/2}$
$I(1,a) = \frac{1}{2}a^{-1}$	$I(3,a) = \frac{1}{2}a^{-2}$	$I(5,a) = a^{-3}$

Z betragtes som en funktion af X og Y ($Z = Z(X,Y)$):

$$dZ = \left(\frac{\partial Z}{\partial X}\right)_Y dX + \left(\frac{\partial Z}{\partial Y}\right)_X dY \qquad \text{[Total differential]}$$

$$\left(\frac{\partial Z}{\partial X}\right)_Y = \left[\left(\frac{\partial X}{\partial Z}\right)_Y\right]^{-1} \qquad \text{[Den inverse af den afledte]}$$

$$\left(\frac{\partial Z}{\partial X}\right)_Y = \left(\frac{\partial Z}{\partial Q}\right)_Y \left(\frac{\partial Q}{\partial X}\right)_Y \qquad \text{[Kædereglen]}$$

$$-1 = \left(\frac{\partial Z}{\partial X}\right)_Y \left(\frac{\partial X}{\partial Y}\right)_Z \left(\frac{\partial Y}{\partial Z}\right)_X \qquad \text{[“Den falske kæderegel”]}$$

$$\left(\frac{\partial}{\partial X}\left(\frac{\partial Z}{\partial Y}\right)_X\right)_Y = \left(\frac{\partial}{\partial Y}\left(\frac{\partial Z}{\partial X}\right)_Y\right)_X \qquad \text{[Ombytning af diff. rækkefølge]}$$

5 Konstanter

$k_B = 1.381 \cdot 10^{-23}$ J/K $= 8.617 \cdot 10^{-5}$ eV/K $\qquad R = 8.315$ J/(mol K)
$N_A = 6.022 \cdot 10^{23}$
$h = 6.626 \cdot 10^{-34}$ J s $= 4.136 \cdot 10^{-15}$ eV s
1 atm $= 1.013$ bar $= 1.013 \cdot 10^5$ N/m^2 $= 1.013 \cdot 10^5$ Pa
1 eV $= 1.602 \cdot 10^{-19}$ J
1 u $= 1.661 \cdot 10^{-27}$ kg