Exam originally in Danish, translated by Bo Jakobsen 2011

Only problem 1 and 2 is at the moment translated!!!

Written 4-hour exam in
Thermodynamics and Statistical Mechanics
Tuesday 28/6-2011, kl 10.00-14.00

The exam consists of three problems on two pages altogether. The weight of the problems are given, and within a problem sub-questions have the same weight.

Writing aids and a simple calculator (that is one which can **not** do graphs or symbolic manipulations) are allowed. The "Golden sheet" (3 pages) are included with this exam. No further helping aids are allowed.

Problem 1 (35 %)

In the following 3 sub-questions a gas is considered where the entropy is given as:

$$S = Nk_B \left[\ln \left(\frac{V}{NV_Q} \right) + 5/2 \right] \tag{1}$$

N is the number of atoms, V the volume, and V_Q is the "quantum volume" which is only a function of temperature. The energy of the gas is given as $E = 3/2Nk_BT$.

The gas is in all sub-questions considered as being in an outer isolated box, the box is partitioned in different ways in the three sub-questions. Figure 1 illustrates the considered situations.

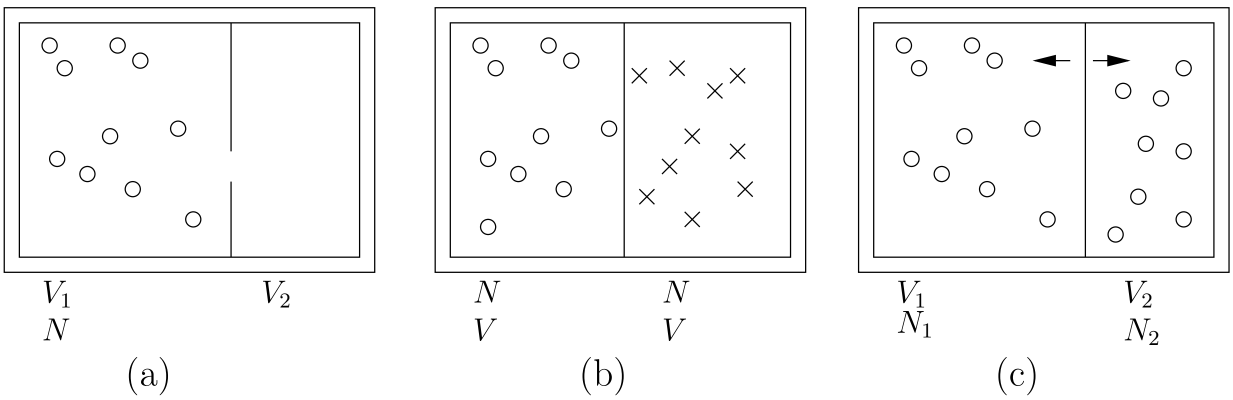

Figure 1. The 3 situations considered in sub-question a–c

Consider firstly a situation where an amount of gas N is in volume V_1 separated from an empty volume V_2 (as sketched on figure 1a.) The gas is now allowed to expand into the empty volume, so that it in the end fills everything.

a) State the change in energy and temperature of the gas, and calculated the change in entropy by the expansion. Comment on the connection between heat and entropy in the process.

Now assume that the gas consists of two different types of atoms, which can be distinguished. There are N atoms of each kind. The total volume can be separated in two equal sized parts by a heat conducting wall. Consider now a situation where we firstly have the two types of atoms separated by the wall, and thereafter a situation where no wall is inserted.

b) Calculated the total entropy in the situation with a wall (as sketched on figure 1b) and in the situation where the wall is removed. Comment on the course of the entropy difference.

Finally, consider a situation where the gas is separated by a wall which can move easily and is heat conducting. The volume of the two pars are called V_1 and V_2 respectively and the number of particles N_1 and N_2 respectively. The situation is sketched on figure 1c.

c) Calculated the ratio between V_1 and V_2 in thermodynamical equilibrium.

Problem 3 (35 %)

Consider a system consisting of N independent atoms on a grid. Each atom can be in 6 states: 2 with energy 0ϵ, 1 with energy 1ϵ and 3 with energy 3ϵ, where ϵ is a positive constant ($\epsilon > 0$).

a) Find Helmholtz free energy as function of T.

b) Find the equilibrium entropy as function of temperature.

c) Calculate the $T = 0$ and $T = \infty$ limits of the entropy, and discuss the results in relation to what is expected in these limits.

Consider now a system consisting of a single atom $N = 1$. A spin B is now assigned to each micro state, such that to the states with energy 0ϵ we have $B = -1$, to the states with energy 1ϵ we have $B = 0$ and to the states with energy 3ϵ we have $B = 1$.

d) Calculate the mean value of B in thermal equilibrium and state the vallues in the $T = 0$ and $T = \infty$ limits. Comment on the limits.

Now assume that the total spin of a system with N atoms can be found as the sum of the spin from the individual atoms,

e) Give the total average spin for N atoms as function of temperature.