Written 4-hour exam in
Thermodynamics and Statistical Mechanics
Monday 31/01-11, 10.00-14.00

The exam consists of three problems on two pages altogether. The weight of the problems are given, and within a problem sub-questions have the same weight.

Writing aids and a simple calculator (that is one which can **not** do graphs or symbolic manipulations) are allowed. The "Golden sheet" (3 pages) are included with this exam. No further helping aids are allowed.

Problem 1 (30 %)

a) Show that the total differential of the entropy can be written as:

$$TdS = VT\alpha_S dP + \frac{C_P}{V\alpha_P} dV. \tag{1}$$

Consider now a quasistatic volume change (from V_1 to V_2) at constant pressure.

b) Show that the applied heat is given as:

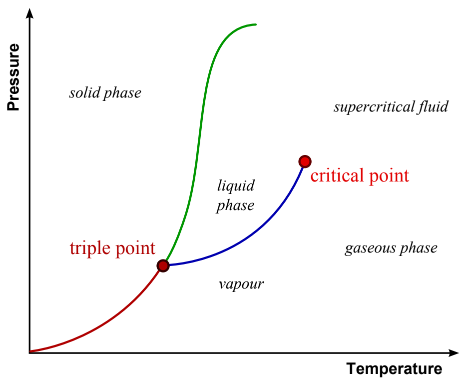
$$Q = \int_{V_1}^{V_2} \frac{C_P}{V\alpha_P} dV \tag{2}$$

and evaluate this integral for a diatomic ideal gas.

c) Calculate the work done on the ideal gas, and the change in thermal energy of the ideal gas.

Problem 2 (30 %)

Consider a system with multiple phases (such as the liquid and gas phases). The figure shows an example of a "phase diagram" for such an system, as function of temperature and pressure.



Example of a (T,P) phase diagram. The diagram shows for a given temperature and pressure which phase is the stable one. The lines which separate the phases are called "phase boundary lines".

a) Which physical quantity determine which phase is the stable one at a given point in the phase diagram, and what is the criterion for the "phase boundary lines"?

b) Show that the slope of the phase boundary lines can be found as:

$$\left.\frac{dP}{dT}\right|_{\text{of phase boundary}} = \frac{L_{\text{total}}}{T\Delta V}. \tag{3}$$

Where L_{total} is the total latent heat associated with the transformation of a given amount of substance and ΔV the change in volume associated with the transformation of the same amount of substance.

Now consider water, which has a *specific* latent heat of $L_{\text{specific}} = 2260\text{J/g}$, and a molar mass of 18g/mol.

c) Calculate a rough estimate of the boiling point of water at the top of Mount Everest, at which the pressure is $\approx 1/3$ the pressure at sea level (which is $1 \cdot 10^5$Pa). You may use the sea level values of L_{total}, T, and ΔV when evaluating the right hand side of eq. (3). (*Hint:* you may consider water vapor as an ideal gas.)

Problem 3 (40 %)

Consider a two state paramagnetic system consisting of N independent dipoles. Each dipole can be in one of two states; one with energy 0 (called the "down state") and one with energy $\epsilon > 0$ (called the "up state").

Consider the system to be in thermal contact with a heat bath with temperature T.

a) Find the Helmholtz free energy as function of T.

b) Find the equilibrium entropy as function of temperature.

c) Calculate the $T = 0$ and $T = \infty$ limits of the entropy, and discuss the results in relation to what is expected in these limits.

Consider now a system consisting of two such dipole systems which are weakly coupled but otherwise *isolated* from the surroundings. Each of the two dipole systems has a total number of dipoles N_1 and N_2 respectively, and a number of dipoles in the up state $n_{\text{up},1}$ and $n_{\text{up},2}$ respectively.

The total energy in the system is $E = \epsilon(n_{\text{up},1} + n_{\text{up},2})$, and you may assume that $N \gg 0$, $n_{\text{up}} \gg 0$ and $n_{\text{down}} \gg 0$ for both dipole systems.

d) Show that the entropy of sub-system 1 is given as:

$$S_1 = k_B[N_1 \ln(N_1) - n_{\text{up},1} \ln(n_{\text{up},1}) - (N_1 - n_{\text{up},1}) \ln(N_1 - n_{\text{up},1})] \tag{4}$$

e) Discuss the criterion for being in equilibrium, and show by explicit calculation that the following holds in equilibrium:

$$\frac{n_{\text{up},1}}{n_{\text{up},2}} = \frac{N_1}{N_2}. \tag{5}$$

(hint: show first that $\frac{\partial n_{\text{up},2}}{\partial n_{\text{up},1}} = -1$)

End of the exam