Exam January 2010

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The course was 7.5 ects points, and only "Golden Sheet" were allowed.

Problem 1 (35%)

a) Show that the "total differential" for the entropy can be written in terms of the isobaric specific heat and the isobaric thermal expansion coefficient, in the following way:

$$dS = \frac{C_P}{T}dT - V\alpha_P dP.$$

We now consider an isothermal reversible (quasistatic) pressure change from P_i to P_f.

b) Show that the supplied heat is given by the isobaric thermal expansion coefficient as: :

$$Q = -T\int_{P_i}^{P_f} V\alpha_P dP,$$

c) Show that the work done is given by the isothermal compressibility as:

$$W = \int_{P_i}^{P_f} VP\kappa_T dP.$$

In the following it is assumed that α_P and κ_T are pressure independent (this is a good approximation for liquids).

Consider a liquid with the following characteristics: $T = 300\text{K}$, $V = 1 \cdot 10^{-5}\text{m}^3$, $\alpha_P = 1.8 \cdot 10^{-4}\text{K}^{-1}$ og $\kappa_T = 4 \cdot 10^{-11}\text{Pa}^{-1}$

The pressure is changed isothermal and reversible from atmospheric pressure to 1000atm.

d) Calculate the supplied heat and the work done during the pressure change. Comment on the result.
(**Hint:** The volume can be assumed constant, in the evaluation of the integrals, as the volume change is very small (due to the small isothermal compressibility)).

Problem 2 (30%)

a) Show that the following relation holds in general, between the adiabatic pressure coefficient, the isochoric specific heat and the isothermal compressibility :

$$\beta_S = \beta_V + \frac{C_V}{TV\beta_V\kappa_T}$$

(**Hint:** You may *potentially* use the following connection: $C_P = C_V + TV\alpha_P\beta_V$)

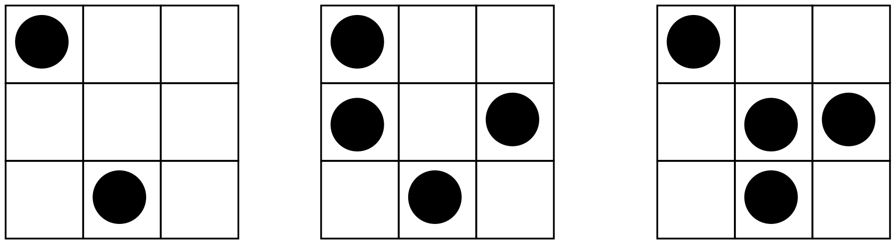
b) Calculate β_S for a diatomic ideal-gas.

Problem 3 (35%)

Consider a model-system with N places and n identical particles
(each position can hold at most one particle).

The energy of the system depends only on the number of particles. Each particle contributes $\epsilon > 0$ energy to the system.

The figure below illustrates three examples.



$N = 9,\ n = 2$ $\quad E = 2\epsilon$ $\qquad$ $N = 9,\ n = 4$ $\quad E = 4\epsilon$ $\qquad$ $N = 9,\ n = 4$ $\quad E = 4\epsilon$

a) Show that the partition function (for any given N and n) in general can be written as:

$$Z_n = \binom{N}{n} e^{-\beta\epsilon n}$$

b) Find a general expression for the Helmholtz free energy for a given N and n.

We now consider the model for fixed size (N) and temperature, but allows the number of particles (n) to vary.
(You may e.g. think of the this in terms of the model being in contact with a particle reservoir, with free exchange of particles between the model and the reservoir — or think of it in terms of "particles" representing objects that can emerge and disappear, e.g. point defects in a crystal).

c) Show that in thermal equilibrium the number of particles are given by:

$$n = \frac{N}{e^{\beta\epsilon} + 1}$$

and calculate $\frac{n}{N}$ for $T \to 0$ og $T \to \infty$.
(**Hint:** you may assume that $N \gg 1$, $n \gg 1$ and $(N - n) \gg 1$.)

d) Give a physical explanation of why the model in general will contain particles in thermal equilibrium, even as the empty model ($n = 0$) at any temperature has the lowest energy (that is $E_{n=0} = 0$).
Furthermore, comment on the found limits for $\frac{n}{N}$.