Exam January 2009

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October 28, 2010

The course was 7.5 ects points, and only "Golden Sheet" were allowed.

Problem 1 (30%)

a)
Show that the following connection is in general true:

$$\beta_S = \frac{C_P}{TV\alpha_P} \tag{1}$$

with β_S the adiabatic (isentropic) pressure coefficient, C_P the isobaric specific heat and α_P the isobaric thermal expansion coefficient.

b)
Consider now a diatomic ideal gas. Derive an expression for β_S as function of P and T for this gas.

c)
The diatomic ideal gas is placed in a container which is thermally isolated from the surroundings. The initial temperature is 30°C. The pressure in the container is now raised slowly by 7%, using a piston. What is the resulting temperature? Would the resulting temperature be greater if it was a mono-atomic ideal gas?

Problem 2 (30%)

a)
Prove the Maxwell relation:

$$\left(\frac{\partial S}{\partial P}\right)_{T,N} = -\left(\frac{\partial V}{\partial T}\right)_{P,N} \tag{2}$$

b)
Prove the formula:

$$P = \frac{k_B T}{Z}\left(\frac{\partial Z}{\partial V}\right)_T \tag{3}$$

(c)
The partition function, Z, is for an ideal gas given by:

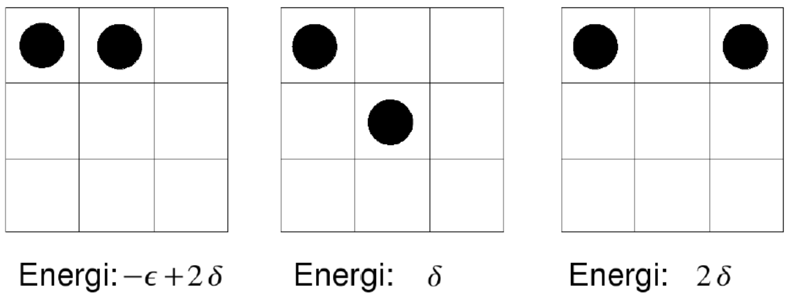
$$Z = \frac{1}{N!}\left(\frac{V Z_{int}}{v_Q}\right)^N, \quad v_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3 \tag{4}$$

show that it satisfies the above equation (3).

Problem 3 (40%)

Consider a system consisting of two identical atoms in a two dimensional grid with 9 places. When the two atoms are nearest neighbors (not on a diagonal) they contribute $-\epsilon$ to the energy of the state. An atom on the edge contributes δ to the energy of the state. No other energy contributions exists.

ϵ is a positive constant. In part a), b), and c) we have $\delta = 0$



a)
Show that the system has 36 states altogether. Show that the system ($\delta = 0$) has 12 states with energy $-\epsilon$.

b)
Calculate the mean energy in thermal equilibrium as function of temperature T ($\delta = 0$).

c)
Find the mean energy in the limit $T \to 0$ and $T \to \infty$, and give a physical interpretation of the result ($\delta = 0$).

d)
Find the entropy in the limits $T \to 0$ and $T \to \infty$ for $\delta = 0$. Find the entropy in the limit $T \to 0$ for $\delta > 0$ and $\delta < 0$ respectively.