Exam January 2008

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The course was 7.5 ects points, and only "Golden Sheet" were allowed.

Problem 1 (40%)

Consider a system consisting of N independent magnetic spins.

They are placed in a magnetic field of size B and each spin has three possible states with energy: $+\mu_B B$, 0, and $-\mu_B B$ respectively (μ_B is the Bohr magnetron).

a) State the partition function, Z, for the total system consisting of N independent spins.

b) Calculate the Helmholts free energy, F, for the system.

c) Calculate the entropy of the system as function of temperature.

d) Find the entropy in the limit $T \to 0$ and the limit $T \to \infty$, give a physical interpretation of the results.

e) We now consider a system identical to the one treated above, except for the energies of the three states of the individual spins, which now is given by $+\mu_B B$, $-\mu_B B$, and $-\mu_B B$ respectively. State the entropy in the limit $T \to 0$.

Problem 2 (20%)

a) Prove the formula:

$$\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}. \tag{1}$$

b) Prove the formula:

$$C_V = \frac{\langle E^2 \rangle - \langle E \rangle^2}{k_B T^2}. \tag{2}$$

Problem 3 (40%)

a) Show that the following general relation holds between the adiabatic thermal expansion coefficient, the isochoric specific heat, and the isochoric pressure coefficient:

$$\alpha_S = \frac{C_V}{TV\beta_V}. \tag{3}$$

We now consider a gas with the following equation of state (a and b are positive constants):

$$P = \frac{Nk_BT}{V - Nb} - a\left(\frac{N}{V}\right)^2. \tag{4}$$

The thermal energy of the gas is given by:

$$U = \frac{f}{2}Nk_BT - Na\frac{N}{V}. \tag{5}$$

b) Calculate the isochoric pressure coefficient, β_V, of the gas.

c) Calculate the adiabatic thermal expansion coefficient, α_S, of the gas.

d) The gas is placed in a container which is thermally isolated from the surroundings. The volume of the container is now slowly increased by 5% utilizing a piston. Give an expression for the resulting temperature change.

e) We now consider the same process as in question **d)**, however the gas is now a diatomic ideal gas. Start temperature is $30°C$. What is the end temperature