Exam January 2006

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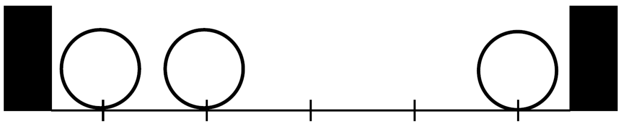
The course was 9 ects points, and the exam an open book exam.

The exam consists of 2 problems.

Problem 1 (50%)

We consider a model where 3 identical atoms are in a one-dimensional grid with 5 grid-positions (there can only be one atom per grid-position).



If two neighbor places in the grid both are occupied by an atom this contributes $-\epsilon$ to the total energy of the state (ϵ is a positive constant).

In the first part of the problem this is the only type of interaction in the model, and the total energy of the sketched state is therefore $-\epsilon$.

a) Show that the system has 3 states with energy -2ϵ, 6 states with energy $-\epsilon$ and one state with energy 0ϵ.

b) Calculate the mean energy in thermal equilibrium as function of temperature.

c) Calculate the entropy of the system in thermal equilibrium as function of temperature.

d) Find the entropy in the limit $T \to 0$ and the limit $T \to \infty$, give an physical interpretation of the results.

The interaction of the atoms with the wall are now included in the model; If an atom is neighbor to a wall this gives a contribution of δ to the total energy of the state (δ is a constant different from 0). The energy of the sketched states is therefore now $-\epsilon + 2\delta$.

e) State the entropy of the system in the limit $T \to 0$ for $\delta > 0$ and $\delta < 0$ respectively.

Problem 2 (50%)

a) Show that the following general relation between the pressure coefficient, the thermal expansion coefficient and the isothermal compressibility holds:

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha_P}{\kappa_T} \tag{1}$$

A system is modeled by the following partition function (where X, b and y are positive constants):

$$Z = \left[X \frac{(V - Nb)}{N} T^{y/2}\right]^N \tag{2}$$

b) Determine the equation of state for this system, and show that the equation of state fulfill equation (1).

c) Show that for this model $\left(\frac{\partial E}{\partial V}\right)_T = 0$, where E is the internal energy of the system in thermal equilibrium.

d) Determine C_P and C_V

e) For the model, determine the adiabatic compressibility, κ_S, as function of P and V. Show that the fund expression for κ_S fulfill the general relation

$$\kappa_S = \kappa_T - \frac{TV\alpha_P^2}{C_P} \tag{3}$$

f) Give a physical interpretation of the model. What is the interpretation of the constants b and y.

Added hints in the translation:
You may use that:

$$C_P = C_V + TV\frac{\alpha_P^2}{\kappa_T} \tag{4}$$