Exam January 2004

Translated by Bo Jakobsen (Autumn 2010)

The course was 9 ects points, and the exam an open book exam.

The exam consists of 2 problems.

Problem 1 (50%)

A system consists of three Ising spins σ_1, σ_2, and σ_3, which each can take the values $+1$ or -1. The system hence have 8 micro states. The energy of system is, in a given state, given by:

$$E = -J(\sigma_1\sigma_2 + \sigma_1\sigma_3 - \sigma_2\sigma_3) + m_B B \sum_i \sigma_i \tag{1}$$

with J a positive constant, m_B is the Bohr magnetron and B is an external magnetic field. In part a) to d) we have $B = 0$.

a) Show that the system has 6 states with energy $-J$, and 2 states with energy $+3J$. Preferably give a physical interpretation of the two types of states.

b) Calculate the energy and specific heat of the system as function of temperature.

c) Calculate the entropy of the system as function of temperature.

d) Find the entropy in the limit $T \to 0$ and in the limit $T \to \infty$, and give a physical interpretation of the results.

e) Now let $B > 0$. State the entropy of the system in the limit $T \to 0$.

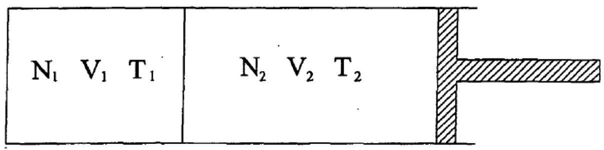
Problem 2 (50%)

We consider a mono-atomic idea-gas.

a) Derive an expression for the isothermal compressibility, κ_T.

b) Show that the adiabatic compressibility is given by $\kappa_S = \frac{3}{5}P^{-1}$.

We now consider a system consisting of two champers, of which one can change its volume by use of a piston:



Each chamber is filed with a mono-atomic idea-gas, and the (fixed) partition between the chambers is a god thermal conductor. N_1 and N_2 are the number of particles in chamber 1 and 2 respectively. V_1 and V_2 are the volume of chamber 1 and 2 respectively. V_1 is constant and V_2 can be changed by the piston. T_1 and T_2 are the temperature in chamber 1 and 2 respectively.

c) We now let the total system be in good thermal contact with the surroundings, and define a isothermal compressibility for chamber 2 (P_2 is the pressure in chamber 2):

$$\kappa_{T,2} \equiv -\frac{1}{V_2}\left(\frac{\partial V_2}{\partial P_2}\right)_T \tag{2}$$

Calculate $\kappa_{T,2}$ and compare to κ_T for the mono-atomic ideal-gas.

d) The total system is now isolated from the surroundings, and we define a adiabatic compressibility for chamber 2:

$$\kappa_{S,2} \equiv -\frac{1}{V_2}\left(\frac{\partial V_2}{\partial P_2}\right)_S \tag{3}$$

(Note that it is the *total* entropy that is kept constant).
Derive an expression for $\kappa_{S,2}$.

e) Give $\kappa_{S,2}$ in the limit of $N_1 \ll N_2$ and in the limit $N_1 \gg N_2$, and give a physical interpretations of the results.