# Simulation project in MatLab

(updated by Nick Bailey)

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Sessions 12 and 13 are dedicated to the simulation project in MatLab in the data room in 13.2. I (probably with some assistance) will be there to help you with your project (in particular, we will make sure you have a working program), but you should expect that some work outside the sessions will be necessary (running the program to get any remaining data you need and writing the report).

The project should be performed in groups of 3-4 people, and the result should be a short, but thorough report, to be handed in in class on Friday, November 6. Besides the report, your m-file should be sent by mail to me at the latest that same day at nbailey@ruc.dk, so I can test your program.

Each group should investigate a problem in classical mechanics. Three suggestions are given:

- Hohmann orbit to Mars
- Resonance between coupled oscillators
- The attack on the Fort

If your group would like to do another project, we can discuss that as well. The structure of project-report should be as follows:

1. Introduction. Briefly summarize the nature of the physical system.
2. Theory. Describe equations selected for the Project. Discuss relevance and limitations of the equations.
3. Method. Describe the algorithm and how it is implemented in the program.
4. Results. Show the results in graphical or tabular form. Discuss results.
5. Analysis. Summarize your results and explain them in simple physical terms whenever possible.

# Hohmann orbit to Mars

It has been proposed to send a spacecraft to Mars by using the so-called Hohmann orbit. We want to examine this possibility by making a MatLab simulation and show that it works and that it is very fuel efficient.

In the Hohmann method, the spacecraft is first placed in a so-called parking orbit around the Earth. The spacecraft is then launched from this orbit and taken to a parking orbit around Mars. The rocket engine delivers the thrust necessary for placing the spacecraft in the parking orbit of Mars. The time when the rocket engines should be turned on depends on the positions of the Earth and Mars.

We consider the spacecraft to be located in a parking orbit around the Earth, but so far away from the Earth, so we can neglect the gravity force from Earth. At some point we increase or decrease the speed of the spacecraft, which will make it leave the parking orbit and start its journey towards Mars, only using the gravitational force of the Sun. When the spacecraft reaches Mars, the speed is again changed, so it will enter a parking orbit around Mars. Fuel is only needed to change the speed while escaping the parking orbit around Earth and entering the parking orbit at Mars.

Analyze the situation, make a model of the spacecraft's coordinates and simulate the model in MatLab. The primary objective of your simulation is to show that it is indeed possible to get from the Earth to Mars by the Hohmann method. Moreover, you should answer the following questions:

- Should the speed of the spacecraft be larger or smaller than the speed of Earth, to land on Mars? How large is the speed difference between Earth and the spacecraft?
- How many days will the journey from Earth to Mars take?
- Which initial angle between Earth and Mars will make the spacecraft land on Mars, and what is the final angle between Earth and Mars?
- When reaching Mars should the spacecraft increase or decrease its speed to enter a parking orbit around Mars? And what is the speed difference between the spacecraft and Mars?
- Does it make a difference to the journey parameters what the mass of the spacecraft is?

**Hints:**

- Calculate only in two dimensions.
- Assume Earth and Mars move in perfect circles (uniform circular motion around the Sun).
- Assume that only the gravitational force from the sun affects the spacecraft.
- All needed astronomical units can be found in the textbook.
- Use the scheme from the MatLab notes on setting up the simulation.
- Carefully consider the direction of the gravitational force.
- Carefully consider your initial conditions.
- Simulate the orbits of Earth and Mars too, as a check on your simulations.
- Additional reading can be found in the book *Elements of Newtonian Mechanics* by J.M. Knudsen and P.G. Hjorth. There is also a Wikipedia article ("Hohmann transfer orbit"), but be careful—do not quote formulas from there unless you also derive them.

# Two coupled oscillators

The mass-spring oscillator, or equivalently the simple pendulum, is a model of all kinds of vibrational motion in science and engineering, from astrophysics to the dynamics of human walking. Connecting two or more oscillators ("coupling" them), on the other hand, is the basis of many models for complex dynamics in science, from lasers to neuron-circuits in the brain to circadian rhythms in animals.

This simulation project is about the coupling of two mass-spring oscillators with equal masses and spring constants. The coupling is via a third spring with a different spring constant.

Experiments involving this and related systems (for example pendula of equal lengths hanging from a common support) show the following result: if we pull one of the masses while holding the other one still, the first will start to oscillate, but after a while it will slow down while the other starts to oscillate, and after another while that eventually dies out while the first one starts up again. This continues (until friction kills everything in the end, but we won't consider friction). This does not happen unless the frequencies of the individual oscillators are equal or very nearly equal to each other (then they are said to be in resonance with one another). Simulate the behavior of the oscillators by analyzing the situation, set up a model, make a simulation of the model and show that we do indeed get the expected behavior. Answer also the following questions: (1) What is the maximum amplitude of the second oscillator when its frequency matches the first? How about when it is a few per cent different? (2) At resonance, how long does it take for the second pendulum to reach maximum amplitude, and how does this depend on the spring constant of the coupling spring? (2) What happens if both pendula are started oscillating together with identical initial displacements and zero velocities? (3) Same question, but opposite displacements (same magnitude)?

## The attack on the fort

A gunship is attacking a fort, which defends a town. The fort is located 1500 m away from the ship and 80 m above the sea level with walls 20 m high. There is a small armory house located 100 m beyond the fort walls. The captain of the gunship knows that a direct hit of the armory house (stuffed with barrels of rum) will force the town to surrender. He also knows that he needs to take air resistance into account when he fires his cannons. He has three different sized cannon balls, with radius 4 cm, 8 cm and 12 cm, made of steel with density 7850 kg/m$^3$. The muzzle speed of the canons for all three is 400 m/s. The density of air is 1.2 kg/m$^3$ and the drag coefficient for a sphere is 0.1.

How would you fire the cannons if you were the captain of the gunship?

Answer this question by analyzing the situation, set up a model, make a simulation of it and show that if the captain follows your strategy, he will make the town surrender. It would be useful to start by considering the case of zero air resistance (see sample problem 4-7 in the book), and then see how much of a difference air resistance makes for the different sized canon balls.